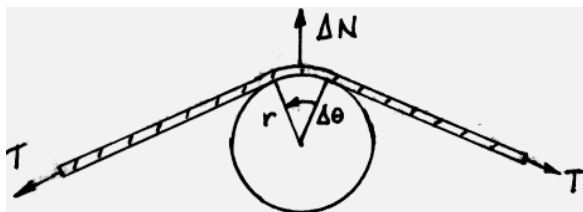


PROBLEM SET 3

1.

- (a.) A clothesline is tied between two poles, 10 m apart, in a way such that the sag is negligible. When a wet shirt with a mass of 0.5 kg is hung at the middle of the line, the midpoint is pulled down by 8 cm. What is the tension in the clothesline?
- (b.) A car is stranded in a ditch, but the driver has a length of rope. The driver knows that he is not strong enough to pull the car out directly. Instead, he ties the rope tightly between the car and a tree that happens to be 50 ft away; he then pushes transversely on the rope at its midpoint. If the midpoint of the rope is displaced transversely by 3 ft when he pushes with a force of 500 N (≈ 50 kg), what force does this exert on the car? If this were sufficient to begin to move the car, and the man pushed the rope another 2 ft, how far would the car be shifted, assuming that the rope does not stretch any further? Does this seem like a practical method of dealing with the situation?

2. A string in tension T is in contact with a circular rod (radius r) over an arc subtending a small angle $\Delta\theta$ (see the figure).



- (a.) Show that the force with which the string presses radially inward on the pulley (and hence the normal force ΔN with which the pulley pushes on the string) is equal to $T\Delta\theta$.
- (b.) Hence show that the normal force per unit length is equal to T/r . This is a sort of pressure which, for a given value of T , gets bigger as r decreases. (This helps to explain why, when a string is tied tightly around

a package, it cuts into the package most deeply as it passes around corners, where r is least.)

- (c.) If the contact is not perfectly smooth, the values of the tension at the two ends of the arc can differ by a certain amount ΔT before slipping occurs. The value of ΔT is equal to $\mu\Delta N$, where μ is the coefficient of friction between string and rod. Deduce from this the exponential relation

$$T(\theta) = T_0 \exp(\mu\theta)$$

where T_0 is the tension applied at one end of an arbitrary arc θ of string and $T(\theta)$ is the tension at the other end.

- (d.) The above result expresses the possibility of withstanding a large tension T in a rope by wrapping the rope around a cylinder, a phenomenon that has been exploited by time immemorial by sailors. Suppose, for example, that the value of μ in the contact between a rope and a (cylindrical) bollard on a dock is 0.2. For $T_0 = 100$ lb applied by the sailor, calculate the values of T corresponding to 1, 2, 3, and 4 complete turns of the rope around the bollard. (It is interesting to note that T is proportional to T_0 . This allows sailors to produce a big pull or not, at will, by having a rope passing around a continuously rotating motor-driven drum. This arrangement can be described as a *force amplifier*).

3. A popular demonstration of inertia involves pulling the tablecloth out from beneath dishes with which the table is set. Suppose a tablecloth just covers the area s^2 of a square table. A dish is in the exact center of the table. The coefficient of sliding friction between the dish and the cloth is μ_1 , and that between the dish and the table is μ_2 . A dinner guest withdraws the cloth swiftly, but at a steady rate. Let the distance the dish moves while in contact with the mov-

ing cloth be x_1 and the distance it moves while in contact with the table be x_2 .

- (a.) Solve for the maximum velocity v of the dish in terms of x_1 , μ_1 , and g .
- (b.) Do the same in terms of x_2 , μ_2 , and g .
- (c.) Show that in order for the dish just to remain on the table,

$$x_1 = (s/2) \frac{\mu_2}{\mu_1 + \mu_2}.$$

- (d.) Find the length of time during which the dish and tablecloth are in contact under conditions (c.).
- (e.) A pitfall for the dinner guest is that the dish may not slide at all, but instead merely move with the cloth. How does she avoid that?

4. A piece of string of length L , which can support a maximum tension T , is used to whirl a particle of mass m in a circular path. What is the maximum speed with which the particle may be whirled if the circle is

- (a.) horizontal;
- (b.) vertical?

5. K&K problem 2.31 “Find the frequency of oscillation...”.

6. K&K problem 2.35 “This problem involves... A block of mass m slides...”.

7. Two skaters (A and B), both of mass 70 kg, are approaching one another, each with a speed of 1 m/sec. Skater A carries a bowling ball with a mass of 10 kg. Both skaters can toss the ball at 5 m/sec relative to themselves. To avoid collision they start tossing the ball back and forth when they are 10 m apart. Is one toss enough? How about two tosses, *i.e.* A gets the ball back? Plot the entire incident on a time *vs.* displacement graph, in which the positions of the skaters are marked along the x axis, and the advance of time is represented by the increasing value of the y axis. (Mark the initial positions of the skaters at $x = +$ or $-$ 5 m, and include the space-time record of the ball’s motion in the diagram.)

This situation serves as a simple model of the standard view of interactions (repulsive, in the present example) between elementary particles.

8. Find the center of mass of a semicircular hoop of radius R .